Title: Systems of Linear Equations – Elimination (Addition) Method (Part 1)
Class: Math 100 or Math 107
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Instructions to tutor: Read instructions and follow all steps for each problem exactly as given.
Keywords/Tags: systems, systems of linear equations, elimination, addition, consistent

Systems of Linear Equations – Elimination (Addition) Method

Purpose:

This is intended to refresh your knowledge about solving systems of linear equations using the elimination (addition) method, where there is a single solution.

Recall that a **system of equations** consists of two or more equations each with two or more variables. A **solution** to a system in two variables is an ordered pair (x, y) that satisfies each equation in the system. For now, we will concentrate on systems of linear equations.

Elimination (Addition) Method – Add a multiple of one equation to the other in order to eliminate one of the variables. After this is done, you will have a single equation with one variable – solve for it. Then back-substitute to find the other.

Example: Solve $\begin{cases} x + \ y = 2 \\ 2x - 3y = 9 \end{cases}$ using the elimination method.

Note that if we multiply the first equation by -2 and add it to the second, the variable x vanishes:

$$\begin{array}{rcrr} -2(x + \ y = 2) & & & -2x - 2y = -4 \\ 2x - 3y = 9 & \Rightarrow & + & 2x - 3y = \ \ 9 \\ \hline & & & -5y = \ \ 5 \end{array}$$

Now that we have eliminated a variable, we may solve for y: $-5y = 5 \quad \Rightarrow \quad y = -1$.

We can substitute this into one of the original equations to find x.

Let's use the first equation: $x + y = 2 \quad \Rightarrow \quad x - 1 = 2 \quad \Rightarrow \quad x = 3$.

So our solution is the ordered pair $(3, -1)$. (Note that this is where the two lines intersect.)

Example: Now it's your turn. Solve $\begin{cases} 3x - 2y = 6 \\ \quad x + 4y = 4 \end{cases}$ using the elimination method.

We have a choice to make here – should we try to eliminate x or y?

What would you have to multiply the 2nd equation by to eliminate x? _____

What would you have to multiply the 1st equation by to eliminate y? _____

Let's take the 2nd option:

$$\begin{array}{rr} & 2(3x - 2y = 6) \\ + & x + 4y = 4 \\ \hline \end{array}$$

Did you find that $x = \frac{16}{7}$? If not, go back and check your work.

Now go back to one of the original equations and solve for y.

Did you get $\left(\frac{16}{7}, \frac{3}{7}\right)$ for your solution? Good! Now try the next two on your own.

Example: Solve using the substitution method.

(a) $\begin{cases} 4x - y = 7 \\ -2x + 3y = 9 \end{cases}$

(b) $\begin{cases} 5x + 6y = 4 \\ 2x - 3y = -3 \end{cases}$

(The answers are $(3, 5)$ for (a) and $\left(-\frac{2}{9}, \frac{23}{27}\right)$ for (b). If you did not get these, consult a tutor for help.)